"Pion laser" phenomenon and other possible signatures of the DCC at RHIC and LHC energies

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The decay of fireballs containing the disoriented chiral condensate (DCC) in A+A collisions has been analyzed. We found that the high phase-space density and probably the large final fireball volume are the favorable factors to extract a DCC signal from the thermal background. Both of these factors are expected to take place at RHIC and LHC energies. A coherent pion component then can be observed in pion spectra, in Bose-Einstein correlations and in distribution of the ratio of neutral to total pions.

I. INTRODUCTION

The main goal of the present and future heavy ion experiments is to discover the new states of the matter associated with chiral and deconfinement phase transitions. The former may be accompanied by the presence of the disoriented chiral condensate (DCC), i.e. correlated region of space-time wherein the quark condensate is chirally rotated from its usual orientation in isotopic space (see, for example, the reviews [1]). The characteristic momentum spread in the DCC is fixed by the inverse size of the region. Recent experimental searches of the DCC signals exploit the overall distribution of the ratio of neutral to total pions without momentum cut as an experimental matter it is not a simple to select low momenta neutral pions. Up to now a clear evidence of the DCC formation has not found. Perhaps, the reason is that the DCC signal is strongly contaminated by the thermal background. Therefore the combine strategy using the analysis of the one- particle inclusive spectra and correlations function of charged pions on soft momenta region could be effective tool to distinguish DCC..

The ability to identify DCC against the background critically depends on the size and phase-space densities of the fireballs. For high enough phase-space densities, which can be reached at RHIC and LHC, the "pion laser" phenomenon could take place. The conception of "pion laser" was introduced in one of the first papers [2] devoted to the problem of symmetrization of pion emission amplitudes at large phase-space densities in finite source. However, the model of independent factorized sources (MIFS) [2–5] leads to the completely chaotic radiation without coherence. The following arguments can explain such a property of the model. The intensive radiation in the MIFS arising at critical value of parameters corresponds to the high-temperature Bose-Einstein (BE) condensation of ideal gas in a finite system [6]. For infinite homogeneous systems, the BE condensate in ideal gas is associated with the phase transition where the coherent condensate wave function can be considered as an order parameter. This phase transition is conditioned by spontaneous breaking a gauge symmetry. To describe it, one should input a fictitious source into the Hamiltonian, go to the thermodynamic limit and *after that* switch off the source [7]. This non-commutative limit procedure leads to the ground state with nonzero expectation value of field, i.e., to the coherent state. However, for spatially inhomogeneous (effectively finite) systems with the finite number of particles, there is no mechanism which results in spontaneous symmetry breaking down. In fact, for finite systems the Hamiltonian of the "pion laser" model has to include an evident symmetry breaking term, for instance, an interaction with a classical source. Then the phenomenon of "pion laser" might be regarded as an effect of weak coherent signal strengthening in dense boson thermal environment. The decay of the DCC is one of the possible mechanisms for classical source to be introduced into the Hamiltonian.

This paper is aimed at finding some experimentally observed consequences of the hypothesis that the DCC can be associated with classical sources interacting with quanta of dense bosonic medium. It is worthy to note that though the whole picture developed is a simplified version of more realistic scenario, it is believed to include some essential features of soft pion production in future experiments at the next generation of heavy ion colliders: RHIC and LHC.

In Section II the space-time evolution of the DCC field is described by the equations of motion of the linear sigma model. Since pions and sigma particles are in a heat bath, the self-consistent Hartree, or mean-field, approximation is appropriate for the model description (see, e.g., [8]). According to Ref. [9], the interaction with the surrounding heat bath is supposed to result in the finite lifetime of classical pion field, and the corresponding decay width is

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introduced. It leads to an additional term, describing the interaction of quasipions with classical source, in the quasiparticle Hamiltonian. Actually, the DCC exists in a finite hot inhomogeneous system that is surrounded by the normal vacuum and hydrodynamically expands with a finite velocity. Thus, it should be stressed that the problem of finding solutions of the self-consistent equations is a very complicated one. Here, we use only some general properties of such solutions.

In Section III the quasiequilibrium stage of the evolution is described by the thermal density matrix for inhomogeneous, effectively finite fireball. The DCC decay in thermal bath can violate the chemical equilibrium for quasipions, and for this reason the corresponding chemical potentials enters the density matrix.

When the system loses the local thermal equilibrium, the description based on the thermal density matrix is no longer adequate. In Section IV we describe the dynamic system evolution at this freeze-out stage by similar form of the Hamiltonian as one at the equilibrium stage (the method is close to [10]). The main parameters of the evolution such as the decay width and the effective time dependent pion masses cannot be, in principle, derived from the self-consistent equations at this stage, they are phenomenologic parameters and differ from themselves at the quasiequilibrium stage. Although we do not consider here other higher order interactions in the system which could result in the BE condensate superfluidity (see, e.g., [11]), we suppose, however, that the freeze-out at high densities can be explained on the basis of the superfluidity of the condensate component of pion liquid.

Results are presented in Section V where signatures of the DCC decay for pion spectra, for HBT correlations and $N_{\pi_0}/N_{\pi_{tot}}$ distribution are analyzed. In Section VI we summarize our results and conclusions.

II. MODEL OF THE DCC DECAY

The linear sigma model describes the O(4) chiral field $\varphi_a = (\sigma, \overrightarrow{\pi})$ by means of the Lagrangian with a simple nonlinear self-interaction

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi^a \partial^\mu \varphi^a - \frac{\lambda}{4}(\varphi^a\varphi^a - v^2)^2 + h\sigma. \tag{1}$$

For the simplicity, let us suppose that in the Hartree approximation there are no cross correlations between different components of the field φ_a. Then the equations of motion for pion "i" component of the order parameter (classical field) $\varphi_{c,i} = \langle \varphi_i \rangle$ and for the field fluctuations (quantum quasiparticles) $\varphi_{q,i}$ can be obtained from the Hartree equation for the pion field $\varphi_i = \varphi_{q,i} + \varphi_{c,i}$ (see, e.g., [12])

$$\partial_\mu \partial^\mu \varphi_i(x) + m_i^2(x)\varphi_i(x) = 2\lambda \left\langle \varphi_i(x) \right\rangle^3, \tag{2}$$

here $m_i^2(x)$ depends on the field averages. Indeed, Eq. (2) can be split into two equations

$$\partial_\mu \partial^\mu \varphi_{c,i} + m_{c,i}^2 \varphi_{c,i} = -f_{c,i}, \quad \partial_\mu \partial^\mu \varphi_{q,i} + m_i^2 \varphi_{q,i} = f_{c,i}, \quad m_{c,i}^2 \equiv m_i^2 - 2\lambda \varphi_{c,i}^2, \tag{3}$$

where $f_{c,i}(x) = 0$ corresponds to the mean-field prescription. Eq. (3) for quasiparticles can be derived from the Hamiltonian $H_{q,i}(t) = \int d^3r \mathcal{H}_{q,i}(x)$,

$$\mathcal{H}_{q,i}(x) = \frac{1}{2}(\pi_{q,i}^2(x) + (\nabla \varphi_{q,i}(x))^2 + m_i^2(x)\varphi_{q,i}^2(x)) - f_{c,i}(x)\varphi_{q,i}(x), \tag{4}$$

where $\pi_{q,i}(x) = \dot{\varphi}_{q,i}(x)$ is the canonically conjugated momentum.

We suppose that in the mean-field approximation the classical field $\varphi_{c,i} = \widetilde{\varphi}_{c,i}(\mathbf{r}, t) = n_i \widetilde{\varphi}_c(\mathbf{r}, t)$ is localized in a hot and dense region and is a quasistationary state (its energy $\omega_c(t)$ changes adiabatically):

$$\widetilde{\varphi}_c(\mathbf{r}, t) = \widetilde{\psi}_c^*(\mathbf{r}, \mathbf{t}) + \widetilde{\psi}_c(\mathbf{r}, \mathbf{t}), \; \widetilde{\psi}_c(\mathbf{r}, \mathbf{t}) = \widetilde{\psi}_c(\mathbf{r}) \exp(-i \int_{t_0}^{t} dt' \omega_c(t')), \tag{5}$$

where $n_i(\theta, \phi)$ is the "i" component of the randomly oriented 3-vector $\mathbf{n}$, $\mathbf{n}^2 = 1$, we suppose that $\mathbf{n}$ does not depend on x. The orientation of the vector $\mathbf{n}$ can be chosen arbitrarily due to the symmetry of the sigma model Lagrangian.

The interaction of the classical field with thermal quasiparticles leads to the finite lifetime of the DCC [9]. After the decay starts, the quantum level is shifted by $\delta\omega$ and the decay width Γ appears in the perturbation theory approximation. Let us neglect $\delta\omega$ and suppose that $\Gamma(t) = \Gamma_1 \Theta(t_f - t) + \Gamma_2 \Theta(t - t_f)$, $\Gamma_1 \ll m_i(x)$, $\Gamma_2 = \Gamma_1$ or $\Gamma_2 \gg \Gamma_1$. As it is discussed later, the last prescription corresponds to a rapid disintegration of the system at $t > t_f$

just below the "thermal freeze-out' temperature $T_f = T(t_f)$. Then taking the decay of the classical field into account, we have:

$$\varphi_{c,i}(x) = \exp(-\frac{1}{2}\int\limits_{t_0}^{t} dt'\Gamma(t'))\widetilde{\varphi}_{c,i}(\mathbf{r},t). \qquad (6)$$

By using the first equation in (3) and (6), one can find the deviation from pure mean field prescription in the case of DCC decay:

$$f_{c,i}(x) = n_i f_c(x) = -\ddot{\varphi}_{c,i}(x) + \exp(-\frac{1}{2}\int\limits_{t_0}^{t} dt'\Gamma(t'))\ \ddot{\widetilde{\varphi}}_{c,i}(x) \neq 0. \qquad (7)$$

III. THERMAL QUASIEQUILIBRIUM STAGE

At the initial stage ($t_0 \le t \le t_f$), the system is in a thermal equilibrium and is described by the quasipion Hamiltonians $H_{q,i}(t)$ (4) and the statistical operator $\rho_{\mathbf{n}}$,

$$\rho_{\mathbf{n}} Sp\rho_{\mathbf{n}} = e^{-\sum_i H^{'}_{q,i}(t)/T},\ H^{'}_{q,i}(t) = \int d^3r[\mathcal{H}_{q,i}(x) - \mu_i \mathcal{J}_{q,i}(x) - \mu'(x)\mathcal{J}'_{q,i}(x)]. \qquad (8)$$

Here μ_i are the chemical potentials which are responsible for violation of the chemical equilibrium and $\mathcal{J}_{q,i}(x) = \varphi^{(+)}_{q,i}(x) \overleftrightarrow{\frac{\partial}{\partial t}} \varphi^{(-)}_{q,i}(x)$ where $\varphi^{(+)}_{q,i}$ and $\varphi^{(-)}_{q,i}$ are the positive and negative field components, respectively. The chemical potentials appear due to the decay of classical field which accompanies the creation of new quasiparticles with their subsequent fast thermalization in the dense environment. To guarantee a finiteness of the system, we include the effective phenomenologic "chemical potential" $\mu'(x)$ in the statistical operator:

$$\mu'(x)\mathcal{J}'_{q,i}(x) = -\frac{1}{2}\varphi^2_{q,i}(x)V(r), \quad V(r) = m^2(t_f)\omega^2 r^2 \equiv \frac{m(t_f)T_f}{R^2}r^2. \qquad (9)$$

Let us suppose that all μ_i are equal to μ and $m_i(x) = m_i(t) \simeq m(t)$. We can disregard the coordinate dependence of masses in Eq. (8) because the "trapping potential" $V(r)$ acts as a strong cut-off factor. Note, however, that $V(r)$ is not real potential: it is not a part of the Hamiltonian $H_{q,i}(t)$. At $\mu = 0$ the quadratic part of $H^{'}_{q,i}$ in Eq. (8) can be diagonalized at $t = t_f$ in the basis $\xi_n(\mathbf{r}) \equiv \xi_{n_1}(x)\xi_{n_2}(y)\xi_{n_3}(z)$,

$$\varepsilon^2_n(t_f)\xi_n(\mathbf{r}) = (-\Delta + V(r) + m^2(t_f))\xi_n(\mathbf{r}), \qquad (10)$$

with the creation and annihilation operators $a^\dagger_{n,i}(t_f)$, $a_{n,i}.(t_f)$, where (see, e.g. [13])

$$\xi_{n_1}(x) = \frac{H_{n_1}(x\sqrt{m(t_{in})\omega})\exp(-x^2 m(t_f)\omega/2)}{((n_1!)2^{n_1}\sqrt{\pi/m(t_f)\omega})^{1/2}}, \qquad (11)$$

$$\varepsilon^2_n(t_f) = 2m(t_f)\omega(n_1 + n_2 + n_3 + \frac{3}{2}) + m^2(t_f). \qquad (12)$$

At the same time the quadratic part of the quasiparticle Hamiltonian $H_{q,i}(t)$ can be diagonalized in the momentum representation with the operators $a_i(\mathbf{k},t_f)$ and $a^\dagger_i(\mathbf{k},t_f)$. The relation between these representations is given by the Bogolyubov transformation with the coefficients $A_n(\mathbf{k})$ and $B_n(\mathbf{k})$:

$$\begin{pmatrix} a_i(\mathbf{k},t_f) \\ a^\dagger_i(\mathbf{k},t_f) \end{pmatrix} = \sum_n \begin{pmatrix} A_n(\mathbf{k}) & B_n(\mathbf{k}) \\ B_n(\mathbf{k}) & A_n(\mathbf{k}) \end{pmatrix} \begin{pmatrix} a_{n,i}(t_f) \\ a^\dagger_{n,i}(t_f) \end{pmatrix}, \qquad (13)$$

$$A_n(\mathbf{k}) = \frac{1}{2}\left(\sqrt{\frac{\varepsilon_k(t_f)}{\varepsilon_n(t_f)}} + \sqrt{\frac{\varepsilon_n(t_f)}{\varepsilon_k(t_f)}}\right)\xi_n(\mathbf{k}), \qquad (14)$$

$$B_n(\mathbf{k}) = \frac{1}{2}\left(\sqrt{\frac{\varepsilon_k(t_f)}{\varepsilon_n(t_f)}} - \sqrt{\frac{\varepsilon_n(t_f)}{\varepsilon_k(t_f)}}\right)\xi_n(-\mathbf{k}). \qquad (15)$$

Eqs. (13) - (15) can be simply derived if the canonical field variables (coordinate and momentum) are written in terms of the creation and annihilation operators in different representations. The coefficients $A_n(\mathbf{k})$ and $B_n(\mathbf{k})$ satisfy to the condition $\sum_n(A_n(\mathbf{k}_1)A_n(\mathbf{k}_2) - B_n(\mathbf{k}_1)B_n(\mathbf{k}_2)) = \delta(\mathbf{k}_1 - \mathbf{k}_2)$. The last relation means that it is a canonical transformation. It is easy to see that the Fock spaces for $a^{\dagger}_{n,i}(t_f)$ and $a^{\dagger}_i(\mathbf{k}, t_f)$ are the unitary nonequivalent representations of the canonical relations. Physical vacuum is the ground state of the Hamiltonian $H_{q,i}(t)$ and is defined as $a_i(\mathbf{k}, t_f)\,|0\rangle = 0$, whereas unphysical vacuum of the "Hamiltonian" $H^{'}_{q,i}$ is defined by $a_{n,i}(t_f)\,|0'\rangle = 0$. So, one should renormalizes the ground state of the statistical operator:

$$\langle ...\rangle_{ren} = \langle ...\rangle - \langle 0'\,|...|\,0'\rangle\,,\;\; \langle ...\rangle = Sp(...\rho). \tag{16}$$

Below we omit the index "ren". At large $R \gg \frac{1}{m(t_f)}\sqrt{\frac{T_f}{m(t_f)}}$ one can calculate the thermal averages of the operators $a(\mathbf{k}, t_f)$, $a^{\dagger}(\mathbf{k}, t_f)$ in the approximation: $A_n(\mathbf{k}) \to \xi_n(\mathbf{k})$, $B_n(\mathbf{k}) \to 0$. Then the thermal averages at given $\mathbf{n}$ in the general case $\mu \neq 0$ are

$$\left\langle a_i^{\dagger}(\mathbf{p},t_f)a_i(\mathbf{p}, t_f)\right\rangle_{\mathbf{n}} = N_{th}(\mathbf{p}) + \left|\langle a_i(\mathbf{p}, t_f)\rangle_{\mathbf{n}}\right|^2, \tag{17}$$

where

$$N_{th}(\mathbf{p}) = \sum_n \frac{\xi_n^2(\mathbf{p})}{\exp(\varepsilon_n - \mu) - 1},\;\; \langle a_i(\mathbf{p}, t_f)\rangle_{\mathbf{n}} = n_i \sum_n g_n \xi_n(\mathbf{p}), \tag{18}$$

and $g_n = \int d^3r \xi_n(\mathbf{r}) f_c(\mathbf{r}, t_f)/((\varepsilon_n - \mu)\sqrt{2\varepsilon_n})$.

IV. FREEZE-OUT STAGE IN THE HEISENBERG REPRESENTATION

At the final stage ($t_f < t \leq t_{out} < \infty$), the system loses the (local) thermal equilibrium, and the misaligned vacuum goes to the normal vacuum with the lifetime $\Gamma_2 \propto (t_{out} - t_f)^{-1}$. Here t_{out} is the "physical asymptotic time" when all interactions are neglected and $\varphi_{c,i}(\mathbf{r}, t_{out}) \simeq 0$, $m_i(\mathbf{r}, t_{out}) \simeq m_{\pi}$, $a_i(\mathbf{p}, t_{out}) \equiv a_{i,out}(\mathbf{p}) \simeq a_{\pi_i}(\mathbf{p}, t_{out})$, the system evolution is determined by the free pions Hamiltonian. We suppose that the evolution of the system during the *continuous* freeze-out stage $t_f < t \leq t_{out}$ is governed by the Hamiltonian (4) with the effective masses $m_i^2(x)$, as is close to [10]. It should be noted that the coordinate dependence of the effective mass in Eq. (4) ensures a finiteness of squeeze-states contributions to spectra. These contributions appear due to the difference in quasiparticles masses at $t = t_f$ and $t = t_{out}$, they are small either in the adiabatic approximation for the mass evolution or when $m_i(\mathbf{r}, t_f) \simeq m_i(\mathbf{r}, t_{out})$. For the sudden freeze-out scenario we will use the second assumption: $m_i(\mathbf{r}, t_f) \simeq m_i(\mathbf{r}, t_{out})$. In these cases we can make a further simplification supposing $m_i(x) = m_i(t) \simeq m(t)$. Then using the Hamiltonian (4), we get the well-known Heisenberg equation's solution for quantum fields which interact with classical source

$$a_{i,out}(\mathbf{p}) \simeq a_i(\mathbf{p},t_f) + n_i d_{\mathbf{p}}(t_{out}), \tag{19}$$

where

$$d_{\mathbf{p}}(t) = \frac{i}{\sqrt{2\varepsilon_{\mathbf{p}}(t_f)}} \int_{t_f}^{t} dt' \exp(i \int_{t_f}^{t'} dt'' \varepsilon_{\mathbf{p}}(t'')) f_c(\mathbf{p}, t'),\;\; \varepsilon_p(t) = \sqrt{\mathbf{p}^2 + m^2(t)} \tag{20}$$

and $f_c(\mathbf{p},t)$ is the Fourier-transformed current $f_c(x)$ in (7).

In the case of the adiabatic freeze-out $\Gamma = \Gamma_1 = \Gamma_2 \ll m(t)$, we have $d_{\mathbf{p}}(t_{out}) \simeq 0$ for all $\mathbf{p}$ if $\omega_c < m$. If $\omega_c \geq m$ then the main contribution has the Breit-Wigner form and reaches its maximal at some momentum $\mathbf{p}$ when $\varepsilon_p(t) \simeq \omega_c(t)$ at some "average" time point $t = \bar{t}$:

$$d_{\mathbf{p}}(t_{out}) \simeq -\frac{\Gamma}{\sqrt{2\varepsilon_{\mathbf{p}}(t_f)}} \frac{\omega_c(\bar{t})\widetilde{\psi}_c(\mathbf{p},t_f)}{i(\varepsilon_{\mathbf{p}}(\bar{t}) - \omega_c(\bar{t})) - \frac{\Gamma}{2}} \exp(-\frac{\Gamma}{2}(t_f - t_0)), \tag{21}$$

The sudden freeze-out case corresponds to $\Gamma_2 \to \infty$. Then we have

$$d_{\mathbf{p}}(t_{out}) \simeq \frac{1}{\sqrt{2\varepsilon_p}}\{\varepsilon_k \varphi_c(\mathbf{p},t_f) + i \,\dot{\varphi}_c\, (-\mathbf{p},t_f)\}, \tag{22}$$

and in terms of pion field operators, Eq. (19) can be rewritten as $a_{\pi_i}(\mathbf{p},t_{out}) \simeq a_{\pi_i}(\mathbf{p},t_f)$.

V. SIGNATURES OF THE DCC DECAY

If we relate two stages through Eqs. (17) and (19), the thermal averages for the operators $a_{i,out}(\mathbf{p})$ at given $\mathbf{n}$ are written as:

$$\left\langle a^{\dagger}_{i,out}(\mathbf{p})a_{i,out}(\mathbf{p})\right\rangle_{\mathbf{n}} = N_{th}(\mathbf{p}) + \left|\langle a_{i,out}(\mathbf{p})\rangle_{\mathbf{n}}\right|^2, \tag{23}$$

where

$$\langle a_{i,out}(\mathbf{p})\rangle_{\mathbf{n}} = n_i(\sum_n g_n\xi_n(\mathbf{p}) + d_{\mathbf{p}}(t_{out})). \tag{24}$$

To evaluate inclusive spectra, we should average over all the orientations $\mathbf{n}$. By taking into account that $n_{+,-} = \frac{\sin\theta}{\sqrt{2}}e^{\pm i\phi}$, $n_0 = \cos\theta$, the one- particle inclusive spectra for π^+, π^-, π^0 pions are

$$N_{\pi^+,\pi^-,\pi^0}(\mathbf{p}) = \frac{1}{4\pi}\int N_{+,-,0}(\mathbf{p})\sin\theta d\theta d\phi = N_{th}(\mathbf{p}) + \frac{1}{3}N_{coh}(\mathbf{p}), \tag{25}$$

where

$$N_{+,-,0}(\mathbf{p}) = N_{th}(\mathbf{p}) + |n_{+,-,0}|^2 N_{coh}(\mathbf{p}),\ N_{coh}(\mathbf{p}) = \left|\sum_n g_n\xi_n(\mathbf{p}) + d_{\mathbf{p}}(t_{out})\right|^2. \tag{26}$$

In order to calculate two-particle inclusive spectra, one has to apply the thermal Wick theorem for fixed $\mathbf{n}$ and then to average over $\mathbf{n}$.

To characterize the relative coherent contribution, we consider the ratio "*signal to noise*":

$$D(\mathbf{p}) = \frac{\frac{1}{3}N_{coh}(\mathbf{p})}{N_{th}(\mathbf{p})}. \tag{27}$$

As it is already mentioned, the classical field is localized in a hot and dense region. The Gaussian length scale of this region for rare gas is $\widetilde{R}^2 \simeq R^2$, while for the dense BE condensate it is $\widetilde{R}^2 \simeq R_c^2 = \frac{R}{2\sqrt{m(t_f)T_f}}$ – which is easily seen from $\xi_0(\mathbf{r})$. Let us suppose for the simplicity that the classical field is localized within the corresponding scale $\widetilde{R}$ with a Gaussian spatial distribution:

$$\widetilde{\psi}_c(\mathbf{r},t_f) = \frac{\kappa}{\widetilde{R}^{3/2}\sqrt{\omega_{c(t_f)}}}\exp(-r^2/4\widetilde{R}^2). \tag{28}$$

If the classical field is large enough at the initial moment t_0 and $\Gamma_1(t_f - t_0) > 1$, the DCC decay at the first thermal stage can lead to an overpopulation of quasiparticles and hence to appearance of the chemical potential $\mu \neq 0$, as it was discussed before. If $\mu \simeq \varepsilon_0$, then the average phase-space density is large enough, $n_{ps} = (N_{coh} + \frac{1}{3}N_{th})/\overline{pr} \gg 1$, and is mainly determined by the coherent condensate contribution

$$n_{ps,coh} \propto \kappa^2\frac{\omega_c}{\varepsilon_0}\left(\frac{\Gamma_1}{(\varepsilon_0 - \mu)}\right)^2 \exp(-\Gamma_1(t_f - t_0)). \tag{29}$$

The ratio *"signal to noise"* can be obtained from (18), (26), (29):

$$\begin{aligned} D(\mathbf{p}) &\simeq \frac{\frac{1}{3}g_0^2\xi_0^2(\mathbf{p})}{\xi_0^2(\mathbf{p})/(\exp(\varepsilon_0 - \mu) - 1)} \propto \\ &\propto \kappa\frac{\Gamma_1}{T_f}\sqrt{\frac{\omega_c n_{ps,coh}}{\varepsilon_0}}\exp(-\frac{\Gamma_1}{2}(t_f - t_0)) \propto \kappa\sqrt{n_{ps}}. \end{aligned} \tag{30}$$

The large phase-space density $n_{ps} \gg 1$ results in the large value $D(\mathbf{p}) \gg 1$ due to the weak signal strengthening in an overpopulated dense boson medium. This is a "pion laser" phenomenon. This effect is supposed to be experimentally observed for pions in the soft energy region: $(p^0 - m_\pi) < R^{-1}\ln n_{ps}$. In pion transverse energy spectra, this effect reveals itself in an essential decrease of the spectrum slope at small energies, $\frac{1}{4m_\pi R_c^2}$, compared to at high energies,

T_f. The same effect observes also in a dense non-coherent condensate [2–4], [6] but the thermal freeze-out of such a dense system can be hardly explained if there are no coherent attributes (e.g., superfluidity).

If the overpopulation at the thermal stage does not arise ($\Gamma_1(t_f - t_0) < 1$) and therefore either $\mu \simeq 0$ or it is far from the critical value $\mu = \varepsilon_0$, the condensate contribution is negligibly small. In the non-relativistic approximation and at sufficiently large volume, $R\sqrt{m(t_f)T} \gg 1$, the thermal part of the radiation can be roughly described (at $R(\varepsilon_0 - \mu)\sqrt{\frac{T}{m(t_f)}} \gg 1$) as the BE gas [6]:

$$N_{th}(\mathbf{p}) \simeq N_{gas}(\mathbf{p}) \equiv \frac{1}{(2\pi)^3} \int \frac{1}{\exp[(\mathbf{p}^2/2m(t_f) + r^2 T_f/2R^2 + \varepsilon_0 - \mu)/T_f] - 1} d^3r. \tag{31}$$

In this case the leading contribution to the coherent emission component $N_{coh}(\mathbf{p})$ is $|d_{\mathbf{p}}(t_{out})|^2$, and $d_{\mathbf{p}}(t_{out})$ reaches its maximum in the case of sudden freeze-out – which we use here as an illustration. From Eqs. (22), (28) and (31), we have

$$D(\mathbf{p}) \simeq \frac{\frac{1}{3}|d_{\mathbf{p}}(t_{out})|^2}{N_{gas}(\mathbf{p})} \propto \kappa^2 \exp(-\Gamma_1(t_f - t_0)) \exp(-\mathbf{p}^2(2R^2 - \frac{1}{2T_f m(t_f)})). \tag{32}$$

Some experimental effects can be observed for pions in the soft energy region: $(p^0 - m_\pi) < (4m_\pi R^2)^{-1}$ which is narrower than for "pion laser". Within this region the inverse of energy spectrum slope, $(4m_\pi R^2)^{-1}$, is considerably less than at high energies, T_f. The interferometry radius slightly decreases in this gas regime [6].

The coherence is most directly connected with the intercept of correlation function (CF). For the CF of $\pi^+\pi^+$-, $\pi^+\pi^-$- pairs the intercepts are

$$C^{++}(\mathbf{p}, \mathbf{0}) = 2 - \frac{4}{5}\left(\frac{D(\mathbf{p})}{1 + D(\mathbf{p})}\right)^2, \ C^{+-}(\mathbf{p}, \mathbf{0}) = 1 + \frac{1}{5}\left(\frac{D(\mathbf{p})}{1 + D(\mathbf{p})}\right)^2. \tag{33}$$

For $D(\mathbf{p}) \to \infty$ one get $C^{++}(\mathbf{p}, \mathbf{0}) = C^{+-}(\mathbf{p}, \mathbf{0}) \to 1.2$, and the effective interferometry radius squared, with

$$C^{++}(\mathbf{p}, \mathbf{q}_{eff}) - 1 = (C^{++}(\mathbf{p}, \mathbf{0}) - 1)e^{--\mathbf{q}_{eff}^2 R_{eff}^2}, \ \mathbf{q}_{eff}^2 R_{eff}^2 = 1, \tag{34}$$

drastically shrinks at condensation [6]:

$$R_{eff}^2(\mathbf{p}) \simeq \frac{R_c^2}{1 + 2\ln(N_{\pi^+}(\mathbf{p})/N_{gas}(\mathbf{p}))}. \tag{35}$$

Finally, let us discuss the problem of the DCC observation where the distribution of the ratio of neutral to total pions $f = N_{\pi_0}/N_{\pi_{tot}}$ is analyzed. It is easy to see from Eq. (26) that the fixed ratio $f = N_{\pi_0}/N_{\pi_{tot}}$ corresponds to an averaging over ϕ at a fixed θ. Then using (26), we get

$$f = \frac{N_{\pi^0,\theta}}{N_{\pi^0,\theta} + N_{\pi^+,\theta} + N_{\pi^-,\theta}} = \frac{\frac{1}{3} + D\cos^2\theta}{1 + D}, \tag{36}$$

where, for example, $N_{\pi^0,\theta} = \int N_{\pi^0,\theta}(\mathbf{p})d^3p$, $N_{\pi^0,\theta}(\mathbf{p}) = \frac{1}{2\pi}\int N_0(\mathbf{p})d\phi$, and $D = \frac{1}{3}\int N_{coh}(\mathbf{p})d^3p/\int N_{th}(\mathbf{p})d^3p \equiv \frac{1}{3}N_{coh}/N_{th}$. Therefore the f-distribution is

$$P(f) = -\sin\theta\frac{d\theta}{df} = \frac{1}{2}\sqrt{\frac{D}{f(1+D) - \frac{1}{3}}}\frac{1+D}{D} \tag{37}$$

in the interval $1/3(1 + D) < f < (1/3 + D)/(1 + D)$, and $P(f) = 0$ beyond the above interval. Note, that neither thermal nor coherent pion number fluctuations does not taken into account in (37). For a weak DCC signal, $D \ll 1$, the distribution is much more narrow than $P(f) = 1/2\sqrt{f}$ at $D \to \infty$. Such a weak signal can be observed if the width of the distribution (37), $D/(1 + D)$, is larger than the width, $\sqrt{2/9N_{\pi_{tot}}}$, of the pure thermal fluctuation distribution peaked around $f = 1/3$. Unfortunately, for rare gas (when the overpopulation at the thermal stage does not arise) one can get: $D \simeq \frac{\frac{1}{3}N_{coh}}{N_{gas}} \simeq \frac{D(\mathbf{0})}{8(R\sqrt{T_f m(t_f)})^3} \ll D(\mathbf{0})$, and so if $D(\mathbf{0})$ is small then the use of $f-$ distributions as signatures of the DCC is impeded. However, at large effective sizes of the system which could be created at RHIC and LHC, such an analysis of the $f-$distribution turns out to be effective for a search for a weak DCC signals. Eqs. (22), (25),

(26) and (28) show that for the rare gas case, the value of $\kappa^2(t_f) = \kappa^2 \exp(-\Gamma_1(t_f - t_0))$ characterizes the number of coherent pions $\frac{1}{3}N_{coh}$ which, when increasing the volume, is expected to be increased also. If $\kappa^2(t_f) \propto R^3 \cdot const$, then D is not decreasing for larger volume while $1/\sqrt{N_\pi}$ is decreasing, for example, $\propto R^{-3/2}$ at constant phase-space densities. This fact can be used to search for a DCC at RHIC and LHC energies even if the phase-space density is not increasing, i.e., if the tendency for the number of pions in the central rapidity area to be approximately proportional to the interferometry volume preserves. One can see also that the increase in $\kappa^2(t_f)$ helps us to distinguish a DCC signal over the thermal pion background from spectra and correlations.

VI. CONCLUSIONS

Some possible experimental consequences of the DCC decay have been analyzed. Our consideration is based on the most general properties of the DCC formation in high energy nucleus-nucleus collisions such as its localization in a hot and dense medium, its dissipation by environment and the finite lifetime, the existences of the freeze-out stage where the (local) thermal equilibrium is destroyed and the misaligned DCC relaxes quickly enough to the normal vacuum. In the picture developed, roughly speaking, a "coherence conservation" low takes place: the large initial classical field (DCC) containing virtually a plenty of coherent pions dissipates partially during the thermal stage, but the resulting overpopulated medium amplifies the residual weak DCC signal ("pion laser" effect) and makes the number of coherent particles to be large again at the freeze-out stage. Such an effect is connected with multiboson effects in the presence of classical source and becomes stronger when the freeze-out phase-space density increases. We have found that the DCC signals in spectra and correlations can be observed in a narrow energy region of soft pions, the width of this region is maximal for the "pion laser" regime. The signal in the distribution of the ratio of neutral to total pions can strongly be contaminated by the thermal background. Nevertheless, even if the tendency of constant phase-space densities preserves for higher energies, a possibility to detect the DCC signal over the thermal background at so large effective volumes as expected in RHIC and LHC heavy ion experiments has been found.

VII. ACKNOWLEDGMENTS

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